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Or (1) is $a^3\rho\frac{vdv}{dx}=g[\delta'(a-x)^3-\rho a^3]$ ....(2).

Integrating (2), $\frac{1}{2}a^3\rho v^2=g[-\frac{1}{4}\delta'(a-x)^4-\rho a^3x]+C$..................(3).

When $x=0$, $v=0$, $\therefore$ $C=\frac{1}{4}(\delta'a^4g)$ and (3) becomes

$\frac{1}{2}(a^3\rho)v^2=g\{(\frac{1}{4}\delta')[a^4-(a-x)^4]-\rho a^3x\}$..............(4).

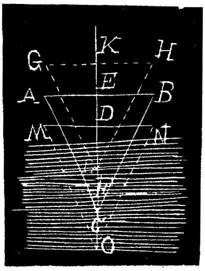
At the greatest height, $v=0$, and then

$(\frac{1}{4}\delta')[a^4-(a-x)^4]-\rho a^3x=0$......................(5),

a biquadratic for x. One value plainly being 0, there remains a cubic to be solved.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $DK=x$, $DL=y$, $\therefore$ $x+y=a$. When the cone is under the liquid its acceleration is $g(\delta'-\delta)/\delta$.



The equation of motion is $d^2x/dt^2=\beta y^3-g$.

When $y=a$, $d^2x/dt^2=g(\delta'-\delta)/\delta$.

$\therefore$ $\beta=\delta'g/a^3\delta$.

$\therefore$ $d^2x/dt^2=(g/a^3\delta)(\delta'y^3-\delta a^3)=(g/a^3\delta)[\delta'(a-x)^3-\delta a^3]$.

$\therefore$ $(dx/dt)^2=-(g/2a^3\delta)[(a-x)^4\delta'+4a^3x\delta]+A$.

When $y=a$, $dx/dt=0$, $x=0$.

$\therefore$ $A=ag\delta'/2\delta$.

$\therefore$ $(dx/dt)^2=v^2=(ag\delta'/2\delta)-(g/2a^3\delta)[\delta'(a-x)^4+4\delta a^3x]$.

When $v=0$, $\delta'(a-x)^4+4\delta a^3x=a^4\delta'$, or

$$x^3-4ax^2+6a^2x=4a^3(\delta'-\delta)/\delta'.$$

This cubic determines x. When $\delta<\frac{1}{4}\delta'$, the cone leaps out of the liquid. At this moment $x=a$, $\therefore$ $v^2=ag(\delta'-4\delta)/2\delta$.

Then $h=v^2/2g=a(\delta'-4\delta)/4\delta$=height of vertex above water.

$\therefore$ Total height of base above liquid$=a+h$.

68. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Find the horizontal and vertical components of the moon's "disturbing force" for any point on the earth's surface making an angle φ with the line joining the center of the earth to the center of the moon.

Solution by the PROPOSER.

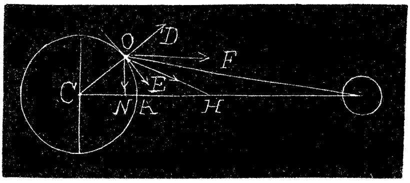
Let CM, the distance from the earth to the moon$=d$; CA, the radius of the earth$=a$; O the point on the earth; $\angle OCA=\varphi$; m=moon's mass; f= "moon's disturbing force" in direction MN; f'=disturbing force in direction ON.

Then $f'=(m/OM^2)\times(ON/OM)=m(ON/OM^3)$.

$ON=a\sin\varphi$, and we can take $OM=d$ without any appreciable error.

$$\therefore f'=\frac{ma\sin\varphi}{d^3}=Q\sin\varphi,$$

where $Q=ma/d^3$.



$$f=m\left(\frac{1}{NM^2}-\frac{1}{CM^2}\right)=m\left(\frac{CM^2-NM^2}{CM^2.NM^2}\right)$$

$$=\frac{mCN(CM+NM)}{CM^2.NM^2}=\frac{m.CN(2d-CN)}{d^2(d-CN)^2}$$

$$=\frac{2m.CN}{d^3}.\frac{(1-CN/2d)}{(1-CN/d)^2}.$$

Since CN/d is small we may neglect it

$$\therefore f=\frac{2m.CN}{d^3}=\frac{2ma\cos\varphi}{d^3}=2\,Q\cos\varphi.$$

Now let f=horizontal, f'=vertical components.

$$\therefore f=f\sin\varphi+f'\cos\varphi=3Q\sin\varphi\cos\varphi=\tfrac{3}{2}Q\sin2\varphi.$$

$$f'=f\cos\varphi-f'\sin\varphi=Q(2\cos^2\varphi-\sin^2\phi)=Q(3\cos^2\varphi-1).$$

DIOPHANTINE ANALYSIS.

68. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

Find a *general* value for p in the expression $4p+1$=the sum of two squares.

I. Solution by J. H. DRUMMOND, LL. D., Portland, Me., and G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa.

Since $4p+1$ is odd one of the squares must be even and the other odd. Take $2q+1$ as one of the numbers and $2s$ for the other, and we have $4q^2+4q+1+4s^2=4p+1$. Hence $p=q^2+q+s^2$ in which q may be zero or any number; and s any number.

II. Solution by the PROPOSER.

From a table in which I have all the odd numbers, up to 12013, that are equivalent to the sum of two squares, I find by inspection that all the values for $4p+1$ can be obtained by making $p=\dfrac{n(n+1)}{2}+\dfrac{a(a+1)}{2}$, in which $n>a$.

Whence $4\left[\dfrac{n(n+1)}{2}+\dfrac{a(a+1)}{2}\right]+1=(n+a+1)^2+(n-a)^2$. (I).

When $a=0$, we have $4\left[\dfrac{n(n+1)}{2}\right]+1=(n+1)^2+n^2$; and we obtain the series of values 5, 13, 25, 41, 61, 85, etc.

When $a=1$, we have $4\left[\dfrac{n(n+1)}{2}+1\right]+1=(n+2)^2+(n-1)^2$; and we obtain the series of values 17, 29, 45, 65, 89, 117, etc.